FYUGP/1st Sem/24(NEP)New

2024

Four Year Under Graduate Programme (FYUGP)

1st Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-101

[Algebra]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer question no. 1 and any *four* from the rest.

1. Answer any *five* questions from the following : 2×5=10

(a) Find the smallest positive integer n such that

$$\left(\frac{1+i}{1-i}\right)^n = 1.$$

(b) Prove that for a complex number z,

$$|z| \geq \frac{1}{\sqrt{2}}(|\operatorname{Re} z| + |\operatorname{Im} z|).$$

P.T.O.

(c) If α, β and γ are the roots of the equation

$$x^3-3x^2+8x-5=0,$$

find the equation whose roots are $2\alpha+3,\ 2\beta+3,\ 2\gamma+3$.

(d) Find the special roots of the equation $x^{15}-1=0$.

(e) For what integral values of m, x^2+x+1 is a factor of $x^{2m}+x^m+1$?

(f) If $a\equiv b \pmod m$ and $c\equiv d \pmod m$, prove that $a+c\equiv b+d \pmod m$, $ac\equiv bd \pmod m$. 1+1

(g) Determine k so that the set $S=\{(k,1,1),(1,k,1),(1,1,k)\}$ is linearly dependent in $\mathbb{R}^3$.

(h) If 0 is an eigenvalue of a square matrix A, is the matrix A singular or non-singular? Give a detailed explanation of your answer.

2. (a) Solve the equation $z^8-z^5+z^3-1=0$. 4

(b) Apply Descarte's rule of signs to examine the nature of the roots of the equation

$$x^6+x^4+x^2+x+3=0.$$ 3

(c) Find the general solution in integers of the equation

$$7x+11y=1.$$ 3

3. (a) If x, y, z are positive real numbers and $x+y+z=1$, prove that

$$8xyz \leq (1-x)(1-y)(1-z) \leq \frac{8}{27}.$$ 5

(b) Solve by Ferrari's method the equation

$$x^4 + 12x^3 + 54x^2 + 96x + 40 = 0.$$ 5

4. (a) Use the theory of congruences to prove that

$7 \mid 2^{5n+3} + 5^{2n+3}$ for all $n \geq 1$. 3

(b) Determine the conditions for which the system of linear equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y - z &= a \\ 5x + 7y + bz &= a^2 \end{aligned}$$

admits (i) a unique solution,

(ii) no solution,

(iii) infinitely many solutions. 2+2+2

(c) Can a cubic equation with real coefficients have complex roots only? 1

5. (a) Determine the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$ 6

P.T.O.

(b) Find the maximum value of $(2x+1)^3(y+2)^2$ when $x+y=3$ and $-\frac{1}{2}<x<5$. 4

6. (a) If $A=\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}$ then show that for every integer $n(\geq 3)$, $A^n=A^{n-2}+A^2-I$. Hence evaluate A^{50}. 4+2

(b) If $\alpha=\cos\frac{2r\pi}{n}+i\sin\frac{2r\pi}{n}$ and if $\gcd(r,n)=1$, $\gcd(p,n)=1$ then prove that

$$1+\alpha^p+\alpha^{2p}+\cdots+\alpha^{(n-1)p}=0.$$ 4

7. (a) Solve the recurrence relation $a_n=3a_{n-1}+2$ subject to the initial condition $a_0=1$. 5

(b) Reduce the real quadratic form

$$Q(x,y,z)=5x^2+y^2+10z^2-4yz-10zx$$

to the normal form. Show that it is positive definite. 4+1

$\dfrac{x}{|\quad|}-\dfrac{y}{|\quad|}=\dfrac{z}{|\quad|}$